Int J Theor Phys (2007) 46: 2617–2625
DOI 10.1007/s10773-007-9376-4

Quasi-resonant Modes of Massive Scalar Fields in Schwarzschild–de Sitter Space-Time

Jia-Feng Chang · Juan Huang · You-Gen Shen

Published online: 15 May 2007

Abstract Recent research of massive fields quasinormal modes suggested that the arbitrary long living modes can be exist. Using different orders of WKB method, we study the massive scalar fields quasinormal modes of Schwarzschild–de Sitter black holes. It is shown that the WKB method can not applied for large massive scalar fields directly in asymptotic flat space-time but can fit well in de Sitter space-time. We prove the non-existence of QRMs in de Sitter space-time and find that the real parts of QNMs increase linearly and the imaginary parts approach to special values as the mass of scalar fields increase.

Keywords Massive Dirac fields · Quasinormal modes · Low-laying

The perturbations of black holes have been studied for many years. It is well-known that there are three stages during the evolution of the fields perturbation in the black holes background: the initial outburst from the source of perturbation, the quasinormal oscillations and the asymptotic tails. The frequencies and damping time of the quasinormal oscillations

J.-F. Chang (✉) · Y.-G. Shen
Shanghai Astronomical Observatory, Chinese Academy of Sciences, Shanghai 200030, China
e-mail: changjf@shao.ac.cn

Y.-G. Shen
e-mail: ygshen@shao.ac.cn

J. Huang
Hefei Institute of Plasma Physics, Chinese Academy of Sciences, Hefei 230031, Anhui, China
e-mail: janejoy@ipp.ac.cn

J.-F. Chang · J. Huang
Graduate School of Chinese Academy of Sciences, Beijing 100039, China

J.-F. Chang · Y.-G. Shen
National Astronomical Observatories, Chinese Academy of Sciences, Beijing 100012, China

Y.-G. Shen
Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100080, China

called "quasinormal modes" (QNMs) are determined only by the black hole's parameters and independent of the initial perturbations [1, 2]. The discovery of the AdS/CFT [3, 4] correspondence and the expanding universe motivated the investigation of QNMs in de Sitter [5, 6] and anti-de Sitter [7–10] space time in the past several years.

Most of the studies on QNMs discuss only on massless fields. For QNMs of massive fields, many authors [11, 12] found that massive modes decay more slowly than massless fields. Ohashi and Sakagami [13] used fraction function method to study the massive QNMs and found the existence of arbitrary long living modes (quasi-resonant modes, QRMs), which means $\mathrm{Im}(\omega \to 0)$ and the perturbation without decay. Konoplya and Zhidenko [14] proved that the purely real frequencies are not forbidden for massive scalar fields and study the overtones QNMs of scalar fields in Schwarzschild black holes using the same method. They also drew a conclusion that no QRMs can exist for massive scalar fields in de Sitter space-time without being numerically proved. In this Letter, we would like to discuss the application of WKB method to evaluate QNMs of the massive scalar and discuss the existence of QRMs for de Sitter space-time. Throughout this paper, we use units in which $G = c = M = 1$.

The metric of Schwarzschild–de Sitter background can be written as

$$\mathrm{d}s^2 = f(r)\mathrm{d}t^2 - \frac{\mathrm{d}r^2}{f(r)} - r^2(\mathrm{d}\theta^2 - \sin^2\theta \mathrm{d}\phi^2), \tag{1}$$

where

$$f(r) = 1 - \frac{2M}{r} - \frac{\Lambda}{3}r^2. \tag{2}$$

M is the mass of the black holes, Λ is the positive cosmological constant. Massive Klein–Gordon equation in curved space-time is

$$\frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^\mu}\left(\sqrt{-g}g^{\mu\nu}\frac{\partial}{\partial x^\nu}\right)\Psi = \mu^2\Psi, \tag{3}$$

here μ is the mass of scalar fields. Setting $\Psi = e^{-i(\omega t - m\phi)}S(\theta)R(r)/r$ and using the tortoise coordinate transformation defined as $\mathrm{d}r^* = \mathrm{d}r/f(r)$, Konoplya [12] derived the radial equation

$$\frac{\mathrm{d}^2R(r)}{\mathrm{d}r_*^2} + [\omega^2 - V]R(r) = 0. \tag{4}$$

Separation constant $\lambda = l(l+1)$, where $l = 0, 1, 2, 3, \ldots$ is the angular momentum quantum number. The effective potential in the one-dimensional Schrödinger-like radial equation (4) is

$$V = f(r)\left(\frac{l(l+1)}{r^2} + \frac{2M}{r^3} + \mu^2\right). \tag{5}$$

In the asymptotic flat space-time, which means $\Lambda = 0$, the effective potential approach constant value μ^2. When the mass of fields exceed the maximum value, the potential barrier change to be a potential step [15]. We take from third order to sixth order WKB method to evaluate the QNMs of massive scalar fields, especially the behaviors of large fields mass. The WKB method for QNMs was used by Schutz and Will [16], which is developed to third order by Iyer and Will [17] and to sixth order by Konoplya [18]. The sixth order WKB method has the form

$$i\frac{\omega^2 - V_0}{\sqrt{-2V_0''}} - \Lambda_2 - \Lambda_3 - \Lambda_4 - \Lambda_5 - \Lambda_6 = n + \frac{1}{2}, \tag{6}$$

where V_0 is the height and the V_0'' is the second derivation with respect to tortoise coordinate of the potential at the maximum, $\Lambda_2 - \Lambda_6$ are presented in [18]. n is the mode number and $n < l$ for low-lying modes which the WKB method can be applied.

Using continued fraction method, Ohashi and Sakagami found the imaginary parts of QNMs reduce to zero as the mass of the scalar fields increase and the existence of QRMs when the fields has special values. In the picture of WKB method, the low-lying QNMs can be regarded as the waves trapped by the peak of the effective potential. Tunneling to occur ω^2 must be smaller than the peak value of the potential and the energies of the fields are always larger than the masses μ, so the low-lying QNMs exist only when $\mu^2 < \omega^2 < V_{\max}$. We can estimate the maximum value $\mu_{\max}$ from [11, 15]:

$$V(r_{\max}, \omega = \mu_{\max}) = (\mu_{\max})^2. \tag{7}$$

The special values of QRMs are about twice as the $\mu_{\max}$ [13]. As the mass of scalar fields increases, the effective potential changes from a single peak potential barrier to a single potential plateau which means the effective potential increases singly. Between these two states, the effective potential has a peak near the black holes horizon and a approach value $\mu^2 > V_0$. The WKB method can do well with $\mu^2 \ll V_0$.

In Fig. 1, we show the imaginary parts of QNMs vary with the mass of scalar fields. It is clear that as the mass of scalar fields increase, the accuracy of WKB method for larger n

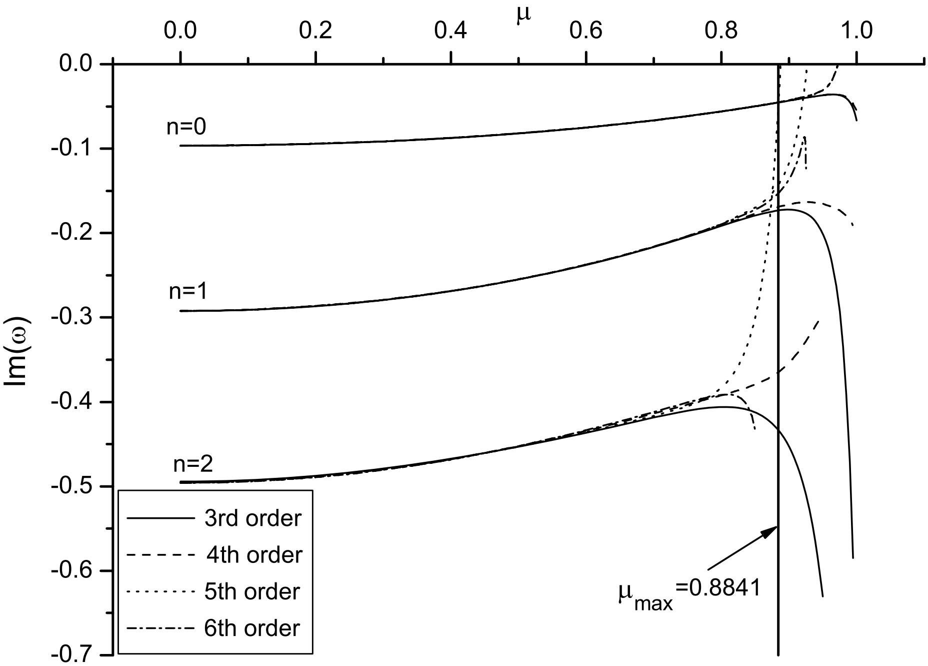

Fig. 1 Variation of imaginary parts of QNMs for massive scalar fields with $l = 3$ and $\Lambda = 0$

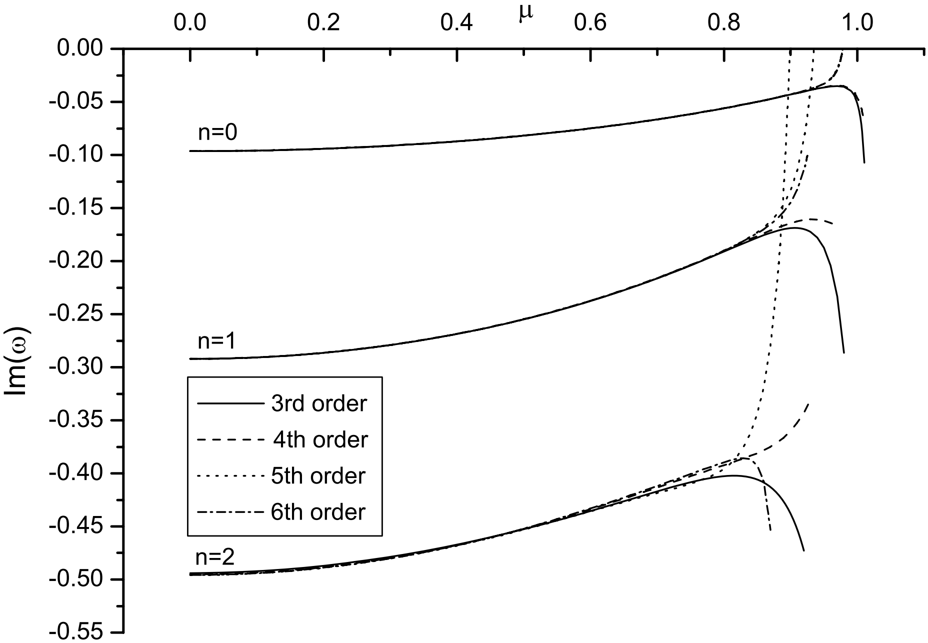

Fig. 2 Im(ω) as a function of μ with $l = 3$ and $\Lambda = 0.0001$

modes lost firstly, which accord with the fact that the WKB method is more accurate for the fundamental modes. When the mass of scalar fields exceed some special values for different mode number n, the differences between the results derived from different orders increase rapidly which means the WKB method out of work.

In the de Sitter space-time, a Schwarzschild black hole has two event horizons: the black holes event horizon $r = r_e$ and the cosmological event horizon $r = r_c$. While r varies from r_e to r_c, the effective potential V reduces to zero, so the potential barrier can not turn effectively into a potential plateau. For the massless scalar fields, the effective potential has a single peak near the black holes horizon. As the mass of scalar fields increases, the effective potential become a double-peak potential barrier. When μ large enough, the potential barrier has a single peak again.

In Figs. 2, 3, 4, 5, 6, 7, 8, 9, we show the imaginary and real parts of QNMs vary with the mass of the scalar fields for the cosmological constant $\Lambda = 0.0001, 0.001, 0.01, 0.05$. In these figures, we think the WKB method can derive the right results when the real and imaginary parts of different orders have the approximately same values. The results suggest that the non-existence of QNMs for massive scalar fields in de Sitter space-time. For $\Lambda = 0.05$ (Figs. 8 and 9), the differences between different orders increase with the mass of scalar fields in reliably degree and decrease later. The differences of different orders vary with μ demonstrate well for $\Lambda = 0.01$ (Figs. 6 and 7). The differences of different orders are large when the peak near the cosmological horizon is equal to or large than another one. The WKB method for higher overtone modes out of work but fit well for fundamental modes in Figs. 6 and 7. When μ exceed special values, the real parts of QNMs increase linearly with μ, and the imaginary parts approach to special values. For the small value of Λ (in Figs. 2, 3, 4, 5), we are not extend to the region where the WKB method can be adopted again, but the results suggest that the imaginary parts of QNMs will approach to a small value which means the

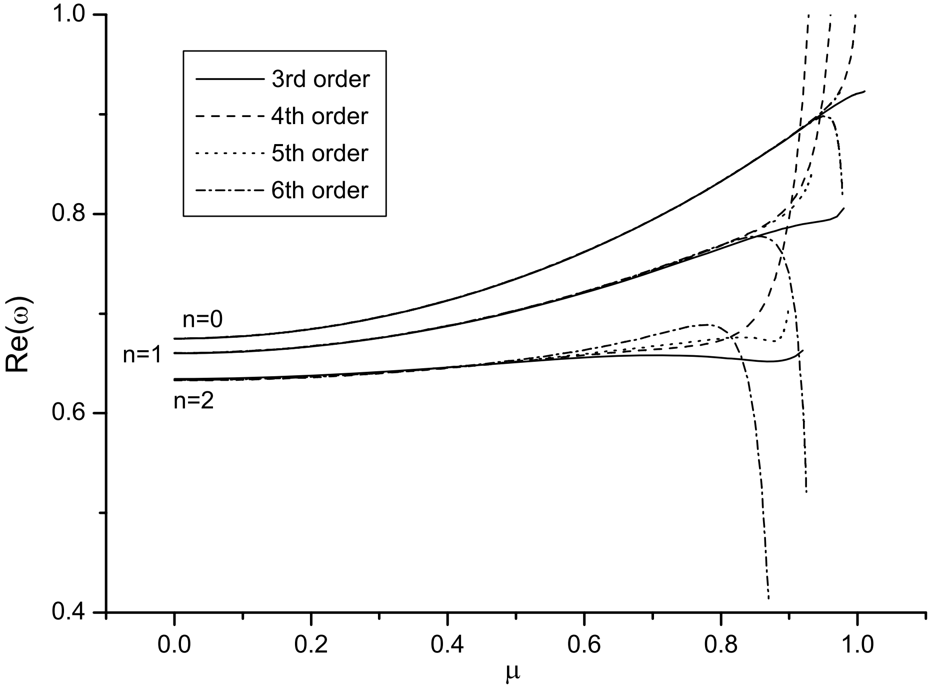

Fig. 3 Re(ω) as a function of μ with $l = 3$ and $\Lambda = 0.0001$

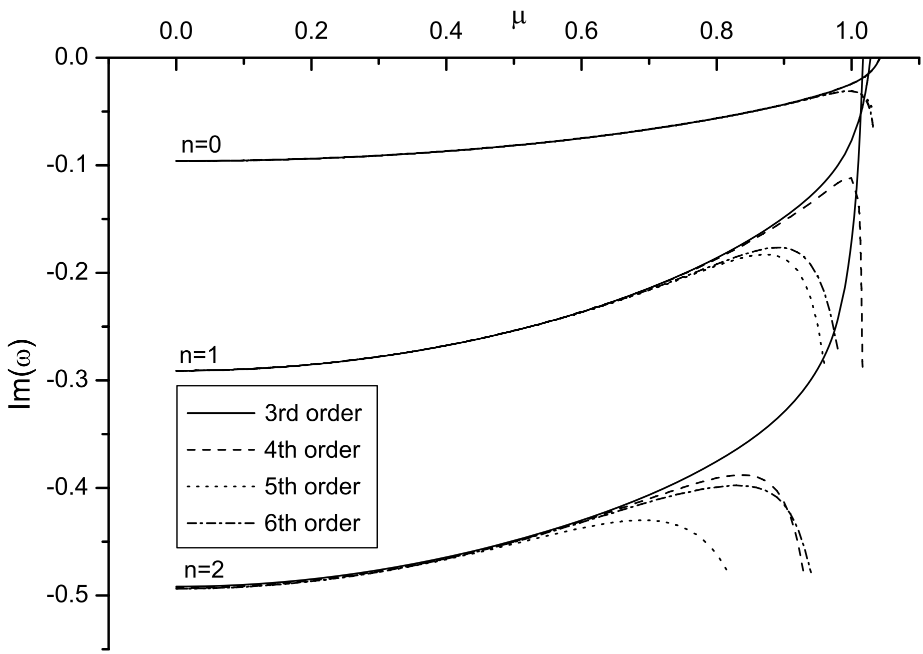

Fig. 4 Im(ω) as a function of μ with $l = 3$ and $\Lambda = 0.001$

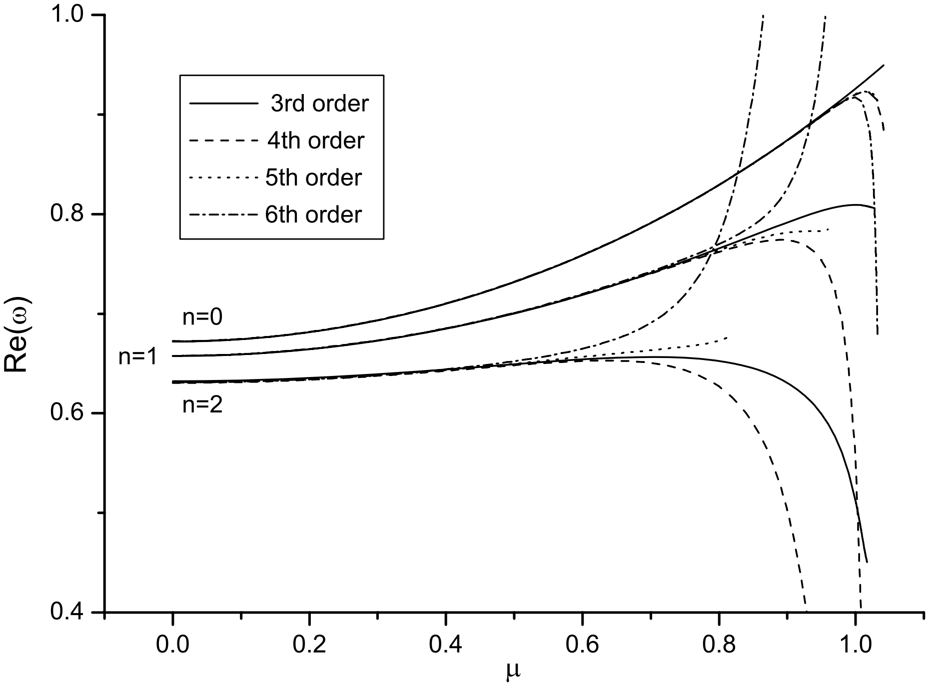

Fig. 5 Re(ω) as a function of μ with $l = 3$ and $\Lambda = 0.001$

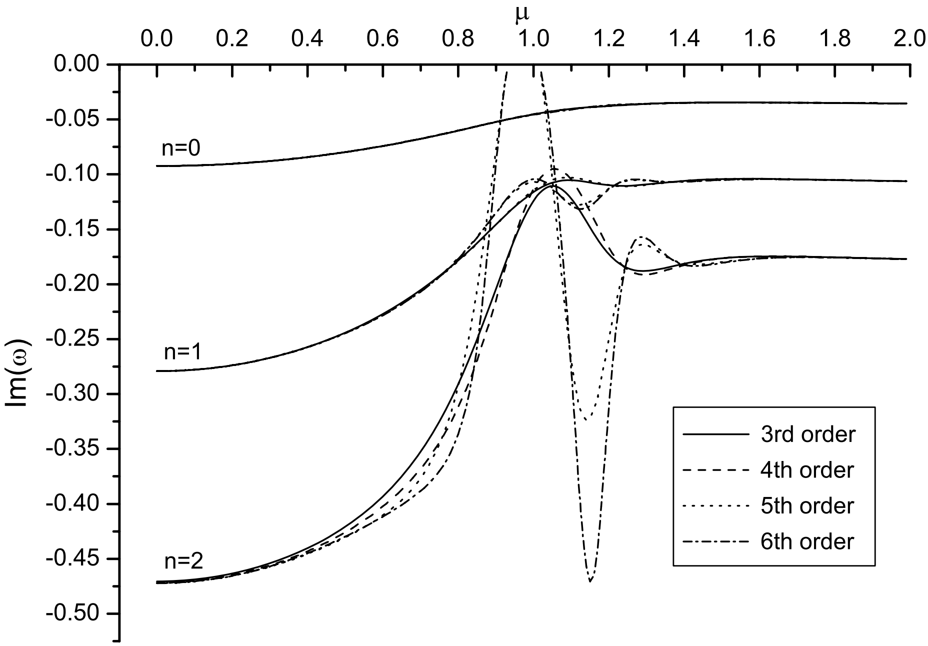

Fig. 6 Im(ω) as a function of μ with $l = 3$ and $\Lambda = 0.01$

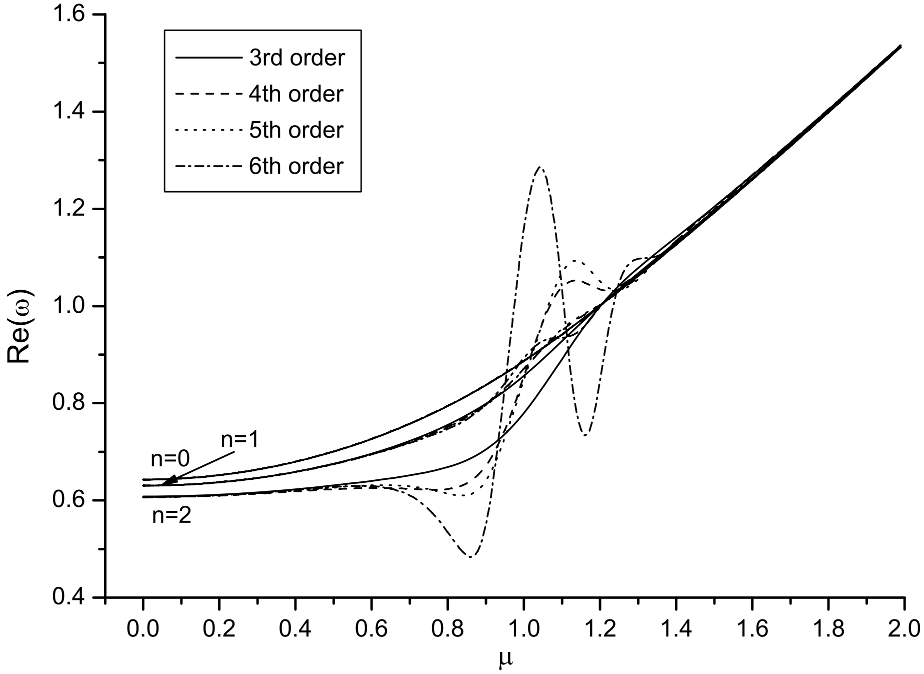

Fig. 7 Re(ω) as a function of μ with $l = 3$ and $\Lambda = 0.01$

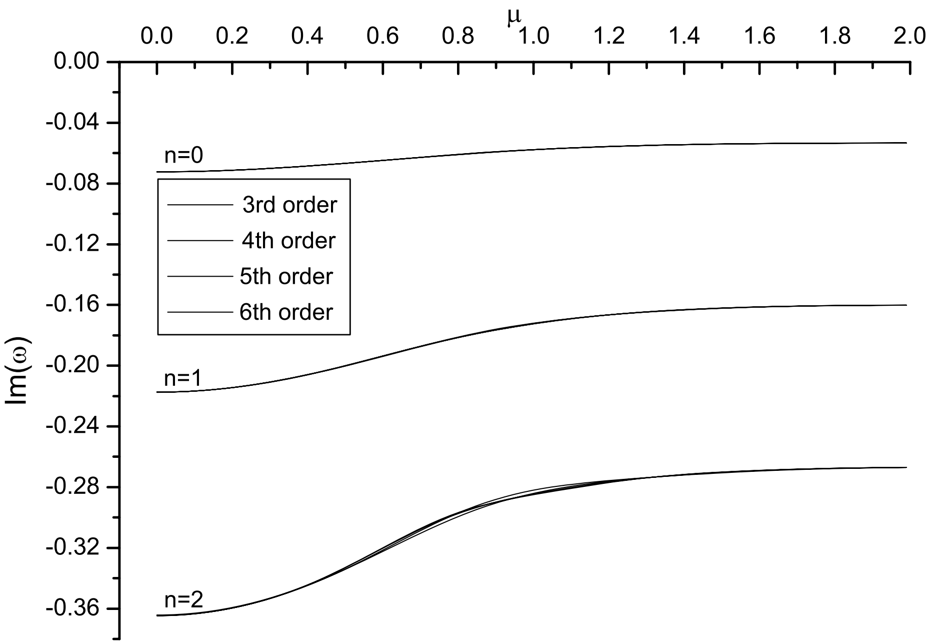

Fig. 8 Im(ω) as a function of μ with $l = 3$ and $\Lambda = 0.05$

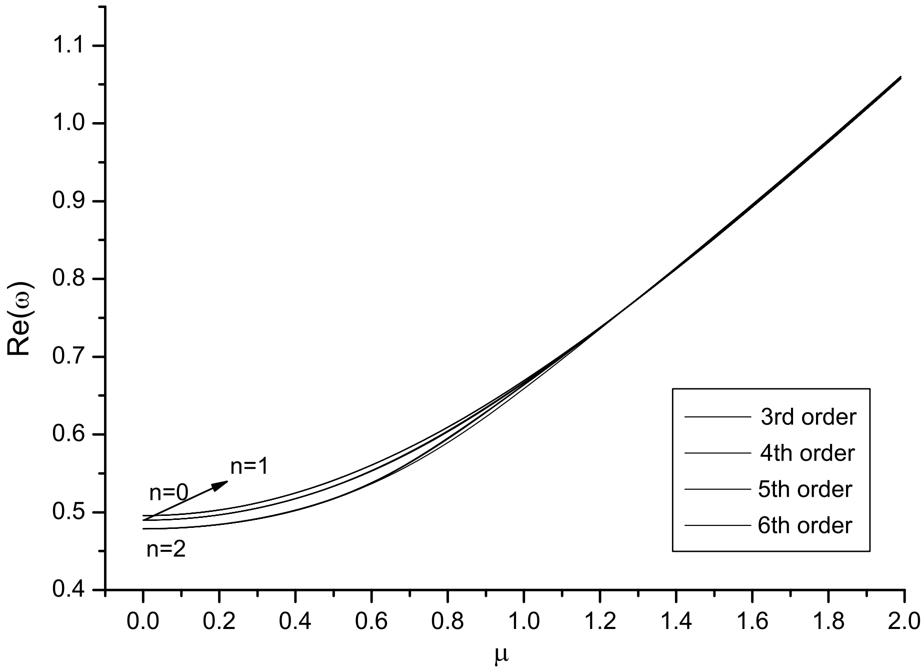

Fig. 9 Re(ω) as a function of μ with $l = 3$ and $\Lambda = 0.05$

non-existence of QRMs for massive scalar fields in de Sitter space-time. In the region where WKB method is out of work, the other methods such as fraction function method will give the accurate results.

In this Letter, we apply third–sixth orders WKB method to massive scalar fields. In the asymptotic flat space-time, the WKB method out of work when $\mu \geq \mu_{\max}$. In the de Sitter space-time, the WKB method can give the accurate especially for large value of Λ. As the mass of scalar fields increases, the real parts of QNMs increase linearly later and the imaginary parts approach to special values.

Acknowledgements The work has been supported by the National Natural Science Foundation of China (Grant No. 10573027, 10663001) and Natural Science Foundation of Shanghai (Grant No. 05ZR14138). One of the authors (J.F. Chang) wishes to thank R.A. Konoplya for supplying sixth WKB corrections printed in MATHEMATICA and X.H. Ge for zealous help during the work.

References

1. Kokkotas, K., Schmidt, B.: Living Rev. Relat. **2**, 2 (1999)
2. Nollert, H.P.: Class. Quantum Gravity **16**, R159 (1999)
3. Birmingham, D., Sachs, I., Solodukhin, S.N.: Phys. Rev. Lett. **88**, 151301 (2002)
4. Cardoso, V., Konoplya, R., Lemos, J.P.S.: Phys. Rev. D **68**, 044024 (2003)
5. Brady, P., Chambers, C., Krivan, W., Laguna, P.: Phys. Rev. D **55**, 7538 (1997)
6. Brady, P., Chambers, C., Laarakkers, W., Poison, E.: Phys. Rev. D **60**, 064003 (1999)
7. Horowitz, D.T., Hubeny, V.E.: Phys. Rev. D **62**, 024027 (2000)
8. Cardoso, V., Lemos, J.P.S.: Phys. Rev. D **63**, 124015 (2001)
9. Cardoso, V., Lemos, J.P.S.: Phys. Rev. D **64**, 084017 (2001)
10. Moss, I.G., Norman, J.P.: Class. Quantum Gravity **19**, 2323 (2002)

11. Simone, L.E., Will, C.M.: Class. Quantum Gravity **9**, 963 (1992)
12. Konoplya, R.A.: Phys. Lett. B **550**, 117 (2002)
13. Ohashi, A., Sakagami, M.: Class. Quantum Gravity **21**, 3973 (2004)
14. Konoplya, R.A., Zhidenko, A.V.: Phys. Lett. B **609**, 377 (2005)
15. Cho, H.T.: Phys. Rev. D **68**, 024003 (2003)
16. Schutz, B.F., Will, C.M.: Astrophys. J. **291**, L33 (1985)
17. Iyer, S., Will, C.M.: Phys. Rev. D **35**, 3621 (1987)
18. Konoplya, R.A.: Phys. Rev. D **68**, 024018 (2003)